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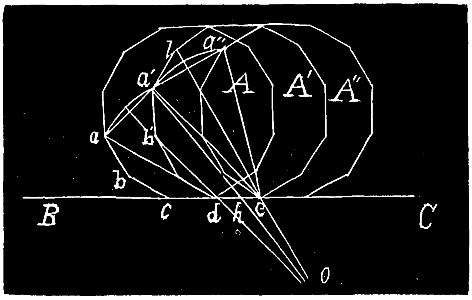
dence with it, any angular point as a, will trace a series of arcs of circles aa', $a'a''$, ... whose centers will be on the line BC. The distance between two consecutive centers will be equal to the side of the polygon. The arc aa' will be described with the radius da; $a'a''$ with the radius ea', ...



The center o of an arc $aa'a''$ which will pass through the points a, a', a'', will be at the intersection of the bisector of the angle ada' and the bisector of the angle $a'ea''$. Since the polygon is inscriptible, the angle $a'el$ is equal to the alternate angle $ea'd$. Therefore, leo is parallel to $a'd$. Similarly, $b'do$ is parallel to $a'e$. Therefore, $a'eod$ is a parallelogram. Its diagonal oa', the radius with which the arc $aa'a''$ is described, is bisected at h.

The above demonstration is applicable to a regular polygon with *any* number of sides. We will now suppose that the number is increased. The length of the side diminishes, and the points d, h, and e approach each other. At the limit, when the polygon becomes a circle, they coalesce, and ha' is the normal. The broken curved line $aa'a''$.......... becomes a cycloid, and the radius of curvature $oa'=2ha'$.

REMARKS ON DEFINITIONS IN TEXT-BOOKS ON GEOMETRY.

By G. W. GREENWOOD, M. A., McKendree College.

What is a circle? A common definition is that it is a plane figure—or a portion of a plane—bounded by a curved line, every point of which is equally distant from a point *within*, called the center. This gives us the impression that a circle is a disc, whereas in more advanced work it is regarded, with other conic sections, merely as a plane curve. It would be better if it were so defined in elementary texts. When we turn, however, to some texts which define it thus, we find something like this: 'A circle is a plane *closed line*, such that all straight lines joining *any* point on this line to a *certain point within* the figure, are equal.'

I shall endeavor to show that such a definition, like the more common one first given, is by no means logical. For this purpose, let us compare them with the following, which, while not altogether free from objections, is, I believe, logical: The locus of points in a plane, at congruent distances from a fixed point in the plane, is a circle. It will be noticed here that nothing is stated or implied concerning the form of the locus or any other properties save the one stated. The word *within* in the earlier definition is unnecessary unless, like a vermiform appendix, it indicates the evolution of this definition from the one first quoted. But in addition to this we see that the definition assumes that the circle divides its plane into at least two parts, one of which is *within*. We are then supplied with other information, such as, that *since* a circle is a *closed curve*, a line from any point *within* to any point *without* the circle *intersects* the circle at least once. This assumes, among other things, that the locus is continuous; yet how all these conclusions are to be deduced from the definition, it would be difficult to state.

Most of the preliminary data in the case of circles, as given in current texts, is based entirely on our visual perception of the figure, with no attempt at deducing it from the definition; none of their off-hand statements are more evident at a glance than the fact that a straight line cannot "intersect" a circle in more than two points, yet a capricious sense of logical rigor requires that this assertion be demonstrated at length.

The premature definition of secants, tangents, tangent circles, etc., in current texts, tends to dull the sense of logical order.

We are not surprised, when the definition given of a circle is similar to one of the two quoted, to find many fallacies in their treatment of these figures. For example, consider the following: 'The line joining a point to the center of a circle is less than, or greater than, a radius, according as the point is within, or without, the circle. Let P be a point within, and Q a point without, a circle whose center is O. *Extend* OP to *meet* the circle in A, and *let* OQ *cut* the circle in C.' These statements are the very facts we wish to prove; namely, that C lies between O and Q, and that A does not lie between O and P.

Generally, with no consideration of the relative magnitudes of arcs of a circle, we are told that a chord subtends two arcs, of which the *lesser* is meant unless otherwise stated. Most of the subject as usually treated is equally vulnerable.

ON KINEMATIC GEOMETRY.—A NEW INVERSOR.

By JOHN J. QUINN, Warren, Pa.

The Theory of Inversion, an important method of investigation especially in many problems of mathematical physics, is one of the distinctive features of modern geometry. It is a method of transformation whereby one surface, or curve, is represented by another surface or curve in a manner that insures similarity in the most minute details. In general, the inverse of a sphere is a sphere; and the inverse of a circle is a circle. In order that two points be in inverse relation it is necessary that they fulfil two conditions:

1. *They must be always collinear with a fixed point.*
2. *The product of their distances from the fixed point must be constant.*

By harnessing this principle it is possible to draw a line that is mathe-